



中山大學
SUN YAT-SEN UNIVERSITY

计算机学院（软件学院）

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING

Compilation Principle

编译原理

第5讲：语法分析(2)

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Review Questions

- Q1: RE to describe $L=\{a^n cb^n\}$, where $0 \leq n \leq 5$?
Yes. RE= $acb | aacbb | aaacbbb | aaaacbbbb | aaaaacbbbbb$
- Q2: is RL applicable to syntax analysis? Why?
No. RL is not powerful enough, e.g., no nested structure
- Q3: input and output of parser?
Input: tokens from lexer; output: parse tree or AST
- Q4: how does grammar relate to syntax?
Grammar is used to specify syntax.
- Q5: productions of grammar?
 $LHS \rightarrow RHS$. e.g., $E \rightarrow E + E$

Syntax Analysis[语法分析]

- Informal description of variable declarations in C[变量声明]
 - Starts with *int* or *float* as the first token[类型]
 - Followed by one or more *identifier* tokens, separated by token *comma*[逗号分隔的标识符]
 - Followed by token *semicolon*[分号]
- To check whether a program is well-formed requires a specification of what is a well-formed program[语法定义]
 - The specification be **precise**[正确]
 - The specification be **complete**[完备]
 - Must cover all the syntactic details of the language
 - The specification must be **convenient**[便捷] to use by both language designer and the implementer
- A **context free grammar** meets these requirements



Example

```
void main(){  
  int a, b, c;  
  if (b == c)  
    return 1;  
}
```

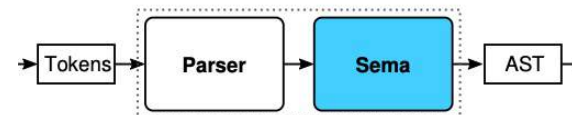
\$clang -cc1 -dump-tokens test.c



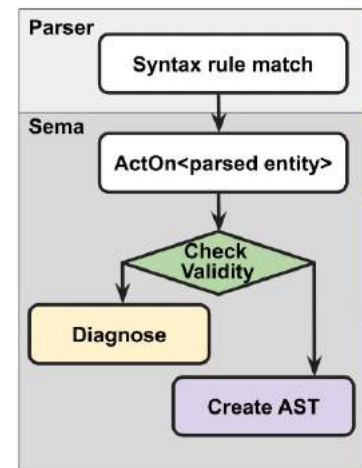
```
void 'void' [StartOfLine] Loc=<parse.c:1:1>  
  identifier 'main' [LeadingSpace] Loc=<parse.c:1:6>  
  l_paren '{' [LeadingSpace] Loc=<parse.c:1:10>  
  r_paren '}' [LeadingSpace] Loc=<parse.c:1:11>  
  l_brace '{' [LeadingSpace] Loc=<parse.c:1:12>  
    int 'int' [StartOfLine] [LeadingSpace] Loc=<parse.c:2:3>  
    identifier 'a' [LeadingSpace] Loc=<parse.c:2:7>  
    comma ',' [LeadingSpace] Loc=<parse.c:2:8>  
    identifier 'b' [LeadingSpace] Loc=<parse.c:2:10>  
    comma ',' [LeadingSpace] Loc=<parse.c:2:11>  
    identifier 'c' [LeadingSpace] Loc=<parse.c:2:13>  
    semi ';' [LeadingSpace] Loc=<parse.c:2:14>  
  if 'if' [StartOfLine] [LeadingSpace] Loc=<parse.c:3:3>  
  l_paren '{' [LeadingSpace] Loc=<parse.c:3:6>  
  identifier 'b' [LeadingSpace] Loc=<parse.c:3:7>  
  equalequal '==' [LeadingSpace] Loc=<parse.c:3:9>  
  identifier 'c' [LeadingSpace] Loc=<parse.c:3:12>  
  r_paren '}' [LeadingSpace] Loc=<parse.c:3:13>  
  return 'return' [StartOfLine] [LeadingSpace] Loc=<parse.c:4:5>  
  numeric_constant '1' [LeadingSpace] Loc=<parse.c:4:12>  
  semi ';' [LeadingSpace] Loc=<parse.c:4:13>  
  r_brace '}' [StartOfLine] Loc=<parse.c:5:1>  
  eof '' [LeadingSpace] Loc=<parse.c:5:2>
```

\$clang -Xclang -ast-dump -fsyntax-only test.c

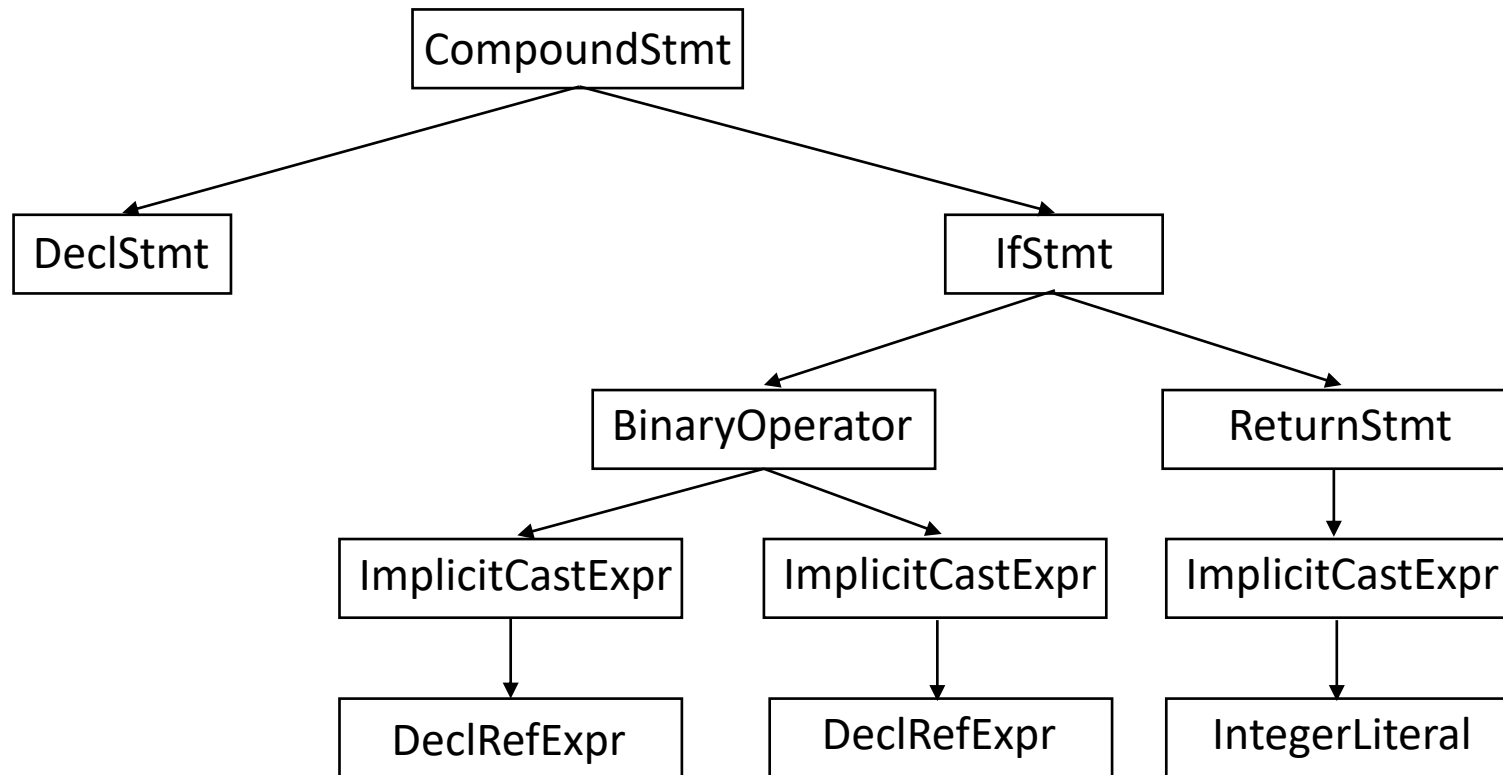
```
-FunctionDecl 0x27999470 <parse.c:1:1, line:5:1> line:1:6 main 'void ()'  
  -CompoundStmt 0x27999800 <col:12, line:5:1>  
    -DeclStmt 0x279996f8 <line:2:3, col:14>  
      -VarDecl 0x27999570 <col:3, col:7> col:7 a 'int'  
      -VarDecl 0x279995f0 <col:3, col:10> col:10 used b 'int'  
      -VarDecl 0x27999670 <col:3, col:13> col:13 used c 'int'  
    -IfStmt 0x279997e8 <line:3:3, line:4:12>  
      -BinaryOperator 0x27999780 <line:3:7, col:12> 'int' '=='  
        -ImplicitCastExpr 0x27999750 <col:7> 'int' <LValueToRValue>  
          -DeclRefExpr 0x27999710 <col:7> 'int' lvalue Var 0x279995f0 'b' 'int'  
        -ImplicitCastExpr 0x27999768 <col:12> 'int' <LValueToRValue>  
          -DeclRefExpr 0x27999730 <col:12> 'int' lvalue Var 0x27999670 'c' 'int'  
      -ReturnStmt 0x279997d8 <line:4:5, col:12>  
        -ImplicitCastExpr 0x279997c0 <col:12> 'void' <ToVoid>  
        -IntegerLiteral 0x279997a0 <col:12> 'int' 1
```



Sema is tight coupling with parser



Example (cont.)



```
void main(){  
    int a, b, c;  
    if (b == c)  
        return 1;  
}
```

```
-FunctionDecl 0x27999470 <parse.c:1:1, line:5:1> line:1:6 main 'void ()'  
-CompoundStmt 0x27999800 <col:12, line:5:1>  
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      -ImplicitCastExpr 0x279997c0 <col:12> 'void' <ToVoid>  
        -IntegerLiteral 0x279997a0 <col:12> 'int' 1
```


Example (cont.)

https://clang.llvm.org/doxygen/ParseStmt_8cpp_source.html

```
case tok::kw_if: // C99 6.8.4.1: if-statement
    return ParseIfStatement(TrailingElseLoc);
    ... ..

StmtResult Parser::ParseIfStatement(SourceLocation *TrailingElseLoc) {
    ... ..
    return Actions.ActOnIfStmt(IfLoc, Kind, LParen, InitStmt.get(), Cond, RParen,
                               ThenStmt.get(), ElseLoc, ElseStmt.get());
}
```

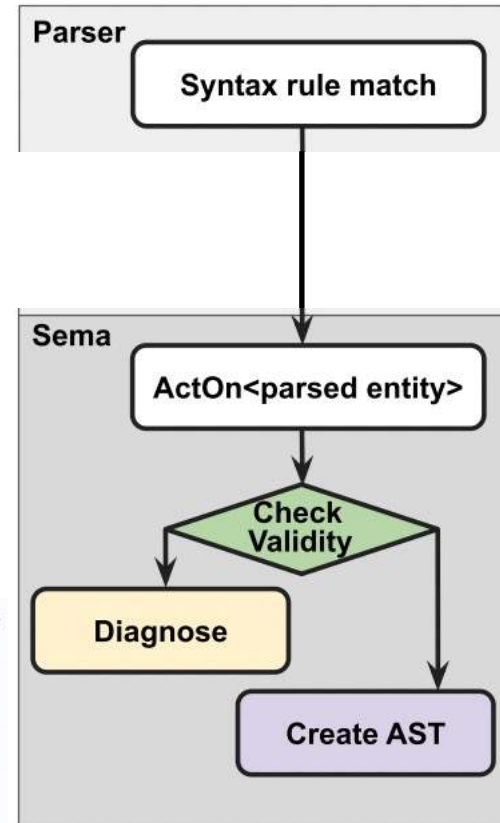
https://clang.llvm.org/doxygen/SemaStmt_8cpp_source.html

```
StmtResult Sema::ActOnIfStmt(SourceLocation IfLoc,
                             IfStatementKind StatementKind,
                             SourceLocation LParenLoc, Stmt *InitStmt,
                             ConditionResult Cond, SourceLocation RParenLoc,
                             Stmt *thenStmt, SourceLocation ElseLoc,
                             Stmt *elseStmt) {
    if (Cond.isInvalid())
        return StmtError();
    ... ..
    return BuildIfStmt(IfLoc, StatementKind, LParenLoc, InitStmt, Cond, RParenLoc,
                       thenStmt, ElseLoc, elseStmt);
}

StmtResult Sema::BuildIfStmt(SourceLocation IfLoc,
                             IfStatementKind StatementKind,
                             SourceLocation LParenLoc, Stmt *InitStmt,
                             ConditionResult Cond, SourceLocation RParenLoc,
                             Stmt *thenStmt, SourceLocation ElseLoc,
                             Stmt *elseStmt) {
    if (Cond.isInvalid())
        return StmtError();

    if (StatementKind != IfStatementKind::Ordinary ||
        isa<ObjCAvailabilityCheckExpr>(Cond.get().second))
        setFunctionHasBranchProtectedScope();

    return IfStmt::Create(Context, IfLoc, StatementKind, InitStmt,
                          Cond.get().first, Cond.get().second, LParenLoc,
                          RParenLoc, thenStmt, ElseLoc, elseStmt);
}
```



Context Free Grammar[上下文无关文法]

- Formal definition[形式化定义]: 4 components **{T, N, s, δ }**
 - T is a finite set of terminals (i.e., token names from lexer)
 - N is a finite set of non-terminals
 - syntactic variables denoting sets of strings, helpful for defining language generated from the grammar
 - S is a special nonterminal (from N) called the start symbol
 - δ is a finite set of production rules of the form such as $A \rightarrow \alpha$, where A is from N and α from $(N \cup T)^*$

- CFG of variable declarations

- {id, int float ;}, {declaration type idlist}, declaration, δ

- Production rules (δ)

declaration \rightarrow *type idlist ;*
idlist \rightarrow *id | idlist , id*
type \rightarrow *int | float*

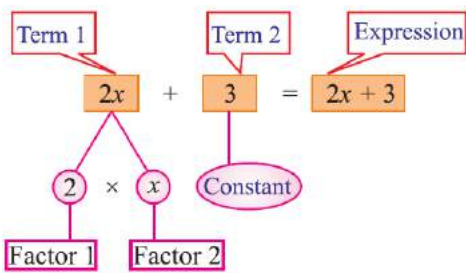
```
void main() {  
    int;  
    int a,;  
    int b, c;  
}
```

Notational Conventions[标识规范]

- These symbols are terminals[终结符]
 - Lowercase letters early in the alphabet, e.g., **a**, **b**, **c**[靠前小写字母]
 - Operator symbols such as **+**, *****, ...[运算符]
 - Punctuation symbols such as **(**, **,** ...[标点符号]
 - Digits **0**, **1**, ..., **9**[数字]
 - Boldface strings such as **id** or **if**, each is a single terminal symbol
- These symbols are non-terminals[非终结符]
 - Uppercase letters early in alphabet, e.g., **A**, **B**, **C**[靠前大写字母]
 - The letter **S**, which, when it appears, is usually the start symbol
 - Lowercase, italic names such as *expr* or *stmt*[小写斜体]
 - When discussing programming constructs, uppercase letters may represent non-terminals for the constructs
 - E.g., **E**: expression[表达式], **T**: term[项], **F**: factor[因子]

Notational Conventions (cont.)

- **Uppercase letters** late in alphabet, e.g., X, Y, Z , represent grammar symbols
 - Either non-terminals or terminals
- **Lowercase letters** late in alphabet, chiefly u, v, \dots, z , represent (possibly empty) strings of terminals
- Lowercase Greek letters, e.g., α, β, γ represent (possibly empty) strings of grammar symbols
 - $A \rightarrow \alpha$
- Unless stated otherwise, the head of the first production is the start symbol[开始符号]



$$\begin{aligned} E &\rightarrow E + T \mid E - T \mid T \\ T &\rightarrow T * F \mid T / F \mid F \\ F &\rightarrow (E) \mid \text{id} \end{aligned}$$

Start symbol: E
Nonterminals: E, T and F
Terminals: everything else

Production Rule and Derivation[推导]

- **Production rule**[产生规则]: $LHS \rightarrow RHS$
 - Aliases[别名]: $LHS \equiv \text{head}$, $RHS \equiv \text{body}$
 - Meaning[含义]: LHS can be constructed (or replaced) with RHS
- **Derivation**[推导]: a series of applications of production rules
 - Replace a non-terminal by the corresponding RHS of a production
- $\beta \Rightarrow \alpha$
 - Meaning: string α is derived from β
 - $\beta \Rightarrow \alpha$: derives in one step
 - $\beta \Rightarrow^* \alpha$: derives in zero or more steps
 - $\beta \Rightarrow^+ \alpha$: derives in one or more steps
- Example: $A \Rightarrow 0A \Rightarrow 00B \Rightarrow 000$
 - $A \Rightarrow^* 000$
 - $A \Rightarrow^+ 000$

Derivation[推导]

- If $S \Rightarrow^* \alpha$, where S is the start symbol of grammar G
- α : **sentential form** of G [句型]
 - A sentential form may contain both terminals and non-terminals (and can be empty)
- α : **sentence** of G [句子]
 - A sentential form with no non-terminals[仅包含终结符]
- **Language**[语言] generated by a grammar
 - $L(G) = \{w: S \Rightarrow^* w, w \in V_T^*\}$
 - A string of terminal w is in $L(G)$, **iff** w is a sentence of G (or $S \Rightarrow^* w$)

S = subject, V = verb, O = object
SV: She laughed.
SVO: She opened the door.

Example

- Grammar $G = \{T, N, s, \delta\}$

- $T = \{0, 1\}$

- $N = \{A, B\}$

- $s = A$

- $\delta = \{ A \rightarrow 0A \mid 1A \mid 0B, B \rightarrow 0 \}$

- Derivation: from grammar to language [文法到语言]

- $A \Rightarrow 0A \Rightarrow 00B \Rightarrow 000$ **Sentence**

- $A \Rightarrow 1A \Rightarrow 10B \Rightarrow 100$

- $A \Rightarrow 0A \Rightarrow 00A \Rightarrow 000B \Rightarrow 0000$

- $A \Rightarrow 0A \Rightarrow 01A \Rightarrow \dots$

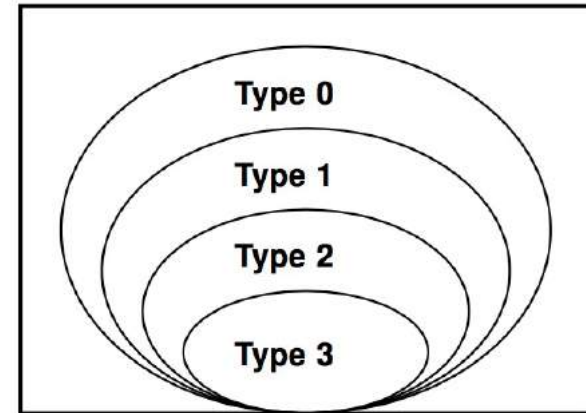
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Sentential form

Language Classification: Chomsky

- **Language classification** based on form of grammar rules
- Four types of grammars:

- Type 0 — unrestricted grammar
 - 0型文法 – 无限制文法
- Type 1 — context sensitive grammar(CSG)
 - 1型文法 – 上下文有关文法
- Type 2 — context free grammar (CFG)
 - 2型文法 – 上下文无关文法
- Type 3 — regular grammar
 - 3型文法 – 正则文法



- Regular Grammar \subseteq CFG \subseteq CSG \subseteq Unrestricted Grammar

Chomsky hierarchy



In 1957, Noam Chomsky published *Syntactic Structures*, an landmark book that defined the so-called Chomsky hierarchy of languages

American linguist, philosopher, cognitive scientist, historian, and activist.

His work has influenced fields such as computer science, mathematics and psychology.

Type 0: Unrestricted Grammar

- Form of rules $\alpha \rightarrow \beta$
 - where $\alpha \in (N \cup T)^+$, $\beta \in (N \cup T)^*$
- Implied restrictions
 - LHS: no ε allowed
- Example:
 - $aB \rightarrow aCD$: LHS is shorter than RHS
 - $aAB \rightarrow aB$: LHS is longer than RHS
 - $A \rightarrow \varepsilon$: ε -productions are allowed
- Derivations
 - Derivation strings may contract and expand repeatedly (since LHS may be longer or shorter than RHS)
 - Unbounded number of productions before target string

Type 1: Context Sensitive Grammar

- Form of rules: $\alpha A \beta \rightarrow \alpha \gamma \beta$
 - where $A \in N$, $\alpha, \beta \in (N \cup T)^*$, $\gamma \in (N \cup T)^+$
- Replace A by γ only if found in the context of α and β
- Implied restrictions
 - LHS: shorter or equal to RHS
 - RHS: no ϵ allowed
- Example:
 - $aAB \rightarrow aCB$: replace A with C when in between a and B
 - $A \rightarrow C$: replace A with C regardless of context
- Derivations
 - Derivation strings may only expand
 - Bounded number of derivations before target string

Type 2: Context Free Grammar

- Form of rules: $A \rightarrow \gamma$
 - where $A \in N$, $\gamma \in (N \cup T)^+$
- Replace A by γ (no context can be specified)
- Implied restrictions
 - LHS: a single non-terminal
 - RHS: no ϵ allowed
 - Sometimes relaxed to simplify grammar but rules can always be rewritten to exclude ϵ -productions
- Example:
 - $A \rightarrow aBc$: replace A with aBc regardless of context

$L = \{ a^n b^n \mid n \geq 0 \}$ is **NOT regular** but **IS a context-free language**.

For the following CFG $G = \langle T, N, S, \delta \rangle$ generates L :

$T = \{ a, b \}$, $N = \{ S \}$ and $\delta = \{ S \rightarrow aSb, S \rightarrow ab \}$

Type 3: Regular Grammar

- Form of rules $A \rightarrow \alpha$, or $A \rightarrow \alpha B$
 - where $A, B \in N$, $\alpha \in T$
- In terms of FA:
 - Move from state A to state B on input α
- Implied restrictions
 - LHS: a single non-terminal
 - RHS: a terminal or a terminal followed by a non-terminal
- Example: $A \rightarrow 1A \mid 0$
 - RE: **1^*0**
- Derivation:
 - Derivation string length increases by 1 at each step

In Practice[实际中]

- Every regular language is a context-free language
 - Context-free languages more general than regular languages
- If PLs are context-sensitive, why use CFGs for parsing?
 - Perfectly suited to describing recursive syntax of expressions and statements
 - CSG parsers are provably inefficient
 - Most PL constructs are context-free
 - if-stmt, declarations
 - The remaining context-sensitive constructs can be analyzed at the semantic analysis stage
 - e.g. def-before-use, matching formal/actual parameters
- In PLs
 - Regular language for lexical analysis
 - Context-free language for syntax analysis

Grammar and Derivation[文法与推导]

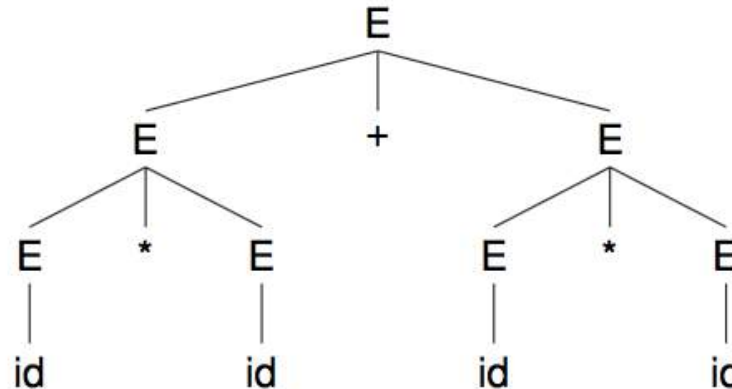
- **Grammar** is used to derive string or construct parser[文法]
- A **derivation** is a sequence of applications of rules[推导]
 - Starting from the **start symbol**
 - $S \Rightarrow \dots \Rightarrow \dots \Rightarrow \dots \Rightarrow$ (sentence)
 - There are choices at each sentential form
 - choice of the nonterminal to be replaced[替换哪个?]
 - choice of a rule corresponding to the nonterminal[怎么替换?]
- Instead of choosing the nonterminal to be replaced, in an arbitrary fashion, it is possible to make an uniform choice at each step[统一化选择替换哪个]
- **Leftmost** and **Rightmost** derivations[最左和最右推导]
 - At each derivation step, **leftmost** derivation always replaces the leftmost non-terminal symbol
 - **Rightmost** derivation always replaces the rightmost one

Example

- Two derivations of string “id * id + id * id” using grammar:
 $E \rightarrow E * E \mid E + E \mid (E) \mid id$
- Leftmost derivation[最左推导]
 - $E \Rightarrow E + E \Rightarrow E * E + E \Rightarrow id * E + E \Rightarrow id * id + E \Rightarrow \dots \Rightarrow id * id + id * id$
- Rightmost derivation[最右推导]
 - $E \Rightarrow E + E \Rightarrow E + E * E \Rightarrow E + E * id \Rightarrow E + id * id \Rightarrow \dots \Rightarrow id * id + id * id$
- Derivations can be summarized as a parse tree[分析树]

Parse Trees[分析树]

- Both previous derivations result in the same parse tree:



Two derivations of string
“id * id + id * id”
using grammar:
 $E \rightarrow E * E \mid E + E \mid (E) \mid id$

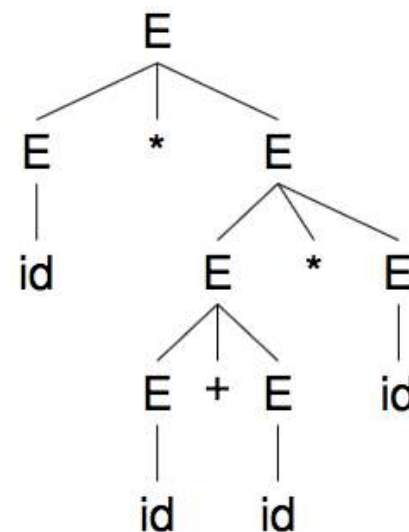
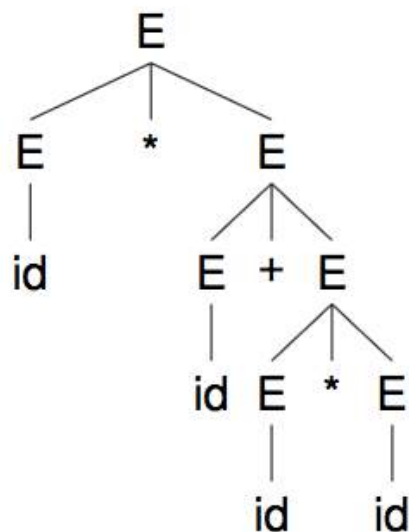
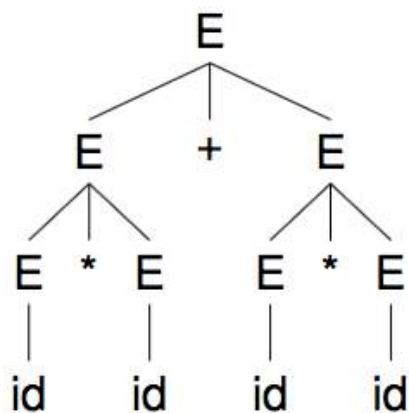
- A **parse tree** is a graphical representation of a derivation
 - But filters out the order in which productions are applied to replace non-terminals[过滤了推导顺序信息]
 - Each **interior node** represents the application of a production
 - Labeled with the non-terminal in the LHS of production
 - **Leaves** are labeled by terminals or non-terminals
 - Constitutes a sentential form (read from left to right)
 - Called the **yield**[产出] or **frontier**[边缘] of the tree

Parse Trees (cont.)

- Derivations and parse trees: **many-to-one** relationship
 - Leftmost derivation order: builds tree left to right
 - Rightmost derivation order: builds tree right to left
 - Different parser implementations choose different orders
 - **One-to-one** relationships between parse trees and either leftmost or rightmost derivations[最左或最右推导与分析树具有一对一对应关系]
- Program structure does not depend on order of rule application, instead it depends on what production rules are applied
 - Grammar must define **unambiguously** set of rules applied

Different Parse Trees

- Grammar $E \rightarrow E * E \mid E + E \mid (E) \mid id$ is ambiguous[二义的]
 - String $id * id + id * id$ can result in 3 parse trees (and more)



The deepest sub-tree is traversed first, thus highest precedence

- Grammar can apply different rules to derive same string
 - Meaning of parse tree 1: $(id * id) + (id * id)$
 - Meaning of parse tree 2: $id * (id + (id * id))$
 - Meaning of parse tree 3: $id * ((id + id) * id)$

Preorder?
 Inorder? ✓
 Postorder?

Ambiguity[二义性]

- Grammar G is **ambiguous** if
 - It produces **more than one parse tree** for some sentence
 - i.e., there exist a string $str \in L(G)$ such that
 - more than one parse tree derives str
 - \equiv more than one leftmost derivation derives str
 - \equiv more than one rightmost derivation derives str
- Unambiguous grammars are preferred for most parsers[文法最好没有歧义性]
 - Ambiguity of the grammar implies that at least some strings in its language have different structures (parse trees)
 - Thus, such a grammar is unlikely to be useful for a programming language, because two structures for the same string (program) implies two different meanings (executable equivalent programs) for this program