



中山大學
SUN YAT-SEN UNIVERSITY

计算机学院（软件学院）

SCHOOL OF COMPUTER SCIENCE AND ENGINEERING

Compilation Principle

编译原理

第6讲：语法分析(3)

张献伟

xianweiz.github.io

DCS290, 3/14/2023



中山大學
SUN YAT-SEN UNIVERSITY



Review Questions

- Grammar G: $stmt \rightarrow \text{if} (expr) stmt \text{ else } stmt$
 $\quad \quad \quad | \text{ while} (expr) stmt \mid v$
 $expr \rightarrow \text{true} \mid \text{false}$
 $N = \{ stmt \ expr \}$
- Is $\text{if} (\text{true}) stmt \text{ else } v$ an sentence of grammar G?
NO. It is a sentential form (句型), as $stmt$ is non-terminal symbol.
- Is $\text{while} (\text{false}) \text{ else } v$ an sentence of G?
NO. It cannot be derived using the production rules.
- Describe the languages generated by G: $list \rightarrow list, id \mid id?$
A list of one or more ids separated by commas.
- How does parse tree relate to derivation?
Parse tree is a graphical representation of derivation. No orders kept.

Ambiguity[二义性]

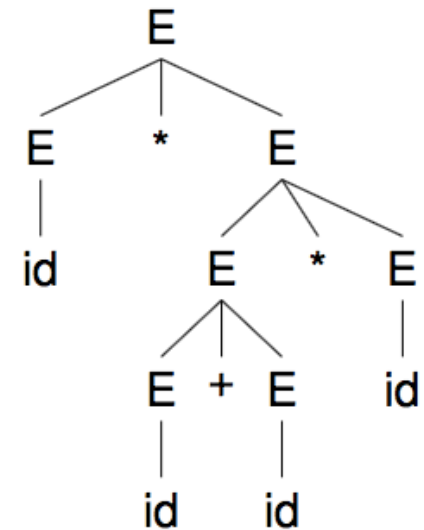
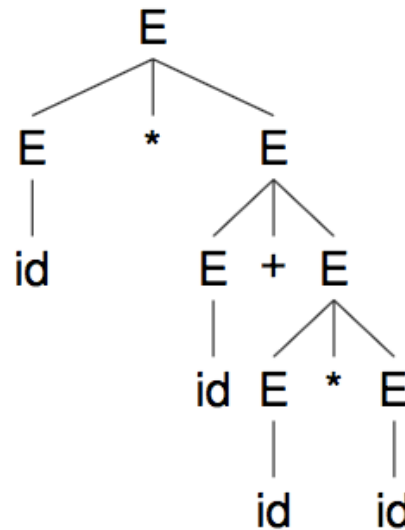
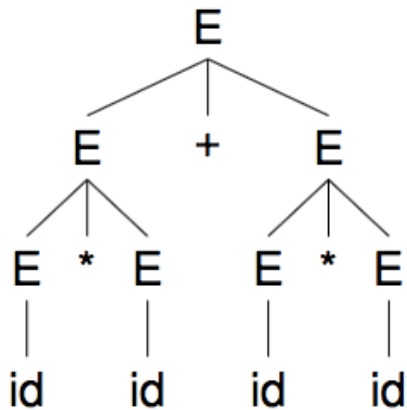
- Grammar G is **ambiguous** if
 - It produces **more than one parse tree** for some sentence
 - i.e., there exist a string $str \in L(G)$ such that
 - more than one parse tree derives str
 - \equiv more than one leftmost derivation derives str
 - \equiv more than one rightmost derivation derives str
- Unambiguous grammars are preferred for most parsers[文法最好没有歧义性]
 - Ambiguity of the grammar implies that at least some strings in its language have different structures (parse trees)
 - Thus, such a grammar is unlikely to be useful for a programming language, because two structures for the same string (program) implies two different meanings (executable equivalent programs) for this program

Ambiguity (cont.)

- Ambiguity is the property of the grammar, not the language
 - Just because G is ambiguous, does not mean $L(G)$ is inherently ambiguous
 - A G' can exist where G' is unambiguous and $L(G') \equiv L(G)$
- Impossible to convert ambiguous to unambiguous grammar automatically[歧义不能自动消除]
 - It is (often) possible to rewrite grammar to remove ambiguity
 - Or, use ambiguous grammar, along with disambiguating rules to “throw away” undesirable parse trees, leaving only one tree for each sentence (as in YACC)
 - A parse tree would be used subsequently for semantic analysis
 - Thus, more than one parse tree would imply several interpretations

Review Ambiguity Example

- Grammar $E \rightarrow E * E \mid E + E \mid (E) \mid id$ is ambiguous[二义的]
 - String $id * id + id * id$ can result in 3 parse trees (and more)



The deepest sub-tree is traversed first, thus highest precedence

- Grammar can apply different rules to derive same string
 - Meaning of parse tree 1: $(id * id) + (id * id)$
 - Meaning of parse tree 2: $id * (id + (id * id))$
 - Meaning of parse tree 3: $id * ((id + id) * id)$

Remove Ambiguity[消除二义性]

- Specify precedence[指定优先级]
 - The higher level of the production, the lower priority of operator
 - The lower level of the production, the higher priority of operator
- Specify associativity[指定结合性]
 - If the operator is left associative, induce left recursion in its production
 - If the operator is right associative, induce right recursion in its production

$E \rightarrow E * E \mid E + E \mid (E) \mid id$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid id$

Possible to get $id + (id + id)$ and $(id + id) + id$

// lowest precedence +
// middle precedence *
// highest precedence ()



$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

Now, can only have more '+' on left

// + is left-associative
// * is left-associative

Remove Ambiguity[消除二义性]

```
int a, b = 1, c = 2;  
a = b = c;  
b = a - b - c;  
b = ???
```

- Specify precedence[指定优先级]
 - The higher level of the production, the lower priority of operator
 - The lower level of the production, the higher priority of operator
- Specify associativity[指定结合性]
 - If the operator is left associative, induce left recursion in its production
 - If the operator is right associative, induce right recursion in its production

$E \rightarrow E * E \mid E + E \mid (E) \mid id$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid id$

Possible to get $id + (id + id)$ and $(id + id) + id$

// lowest precedence +
// middle precedence *
// highest precedence ()



$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$

Now, can only have more '+' on left

// + is left-associative
// * is left-associative

Remove Ambiguity (cont.)

$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid \text{id}$



$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

- String $\text{id} * \text{id} + \text{id} * \text{id}$ can result in
 - Meaning 1: $(\text{id} * \text{id}) + (\text{id} * \text{id})$
 - Meaning 2: $\text{id} * (\text{id} + (\text{id} * \text{id}))$
 - Meaning 3: $\text{id} * ((\text{id} + \text{id}) * \text{id})$

Remove Ambiguity (cont.)

$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid \text{id}$



$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

- String $\text{id} * \text{id} + \text{id} * \text{id}$ can result in
 - Meaning 1: $(\text{id} * \text{id}) + (\text{id} * \text{id})$
 - Meaning 2: $\text{id} * (\text{id} + (\text{id} * \text{id}))$
 - Meaning 3: $\text{id} * ((\text{id} + \text{id}) * \text{id})$

Remove Ambiguity (cont.)

$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$

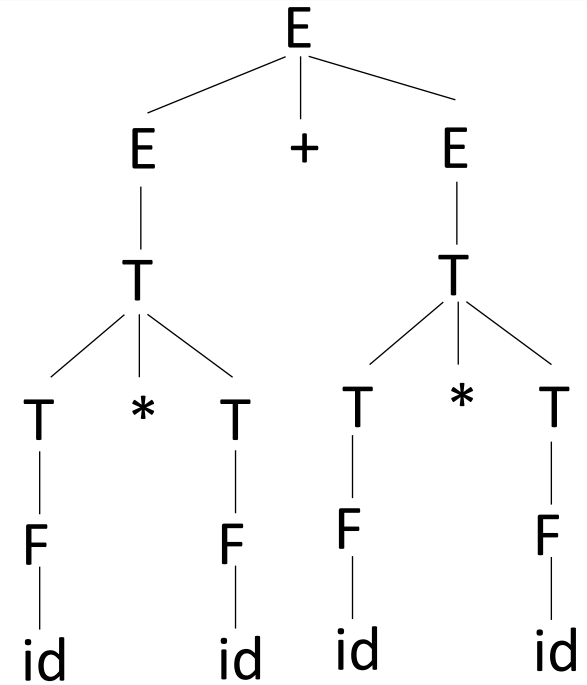


$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid \text{id}$



$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

- String $\text{id} * \text{id} + \text{id} * \text{id}$ can result in
 - Meaning 1: $(\text{id} * \text{id}) + (\text{id} * \text{id})$
 - Meaning 2: $\text{id} * (\text{id} + (\text{id} * \text{id}))$
 - Meaning 3: $\text{id} * ((\text{id} + \text{id}) * \text{id})$



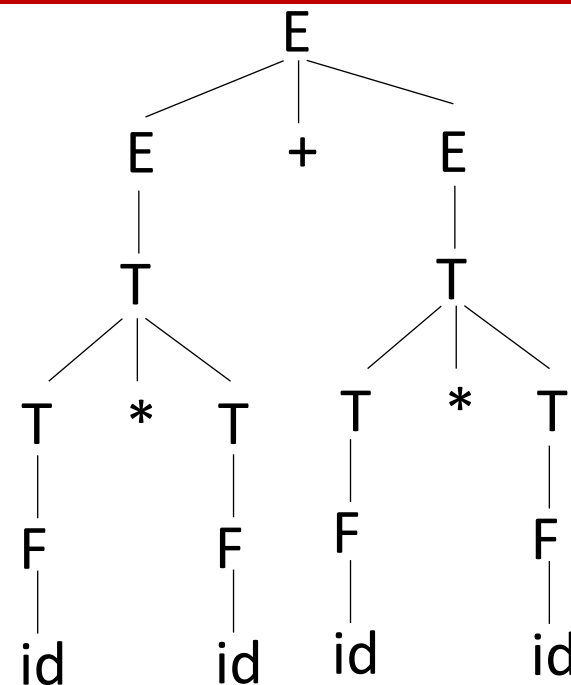
Remove Ambiguity (cont.)

$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$

$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid \text{id}$

$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

- String $\text{id} * \text{id} + \text{id} * \text{id}$ can result in
 - Meaning 1: $(\text{id} * \text{id}) + (\text{id} * \text{id})$ ✓
 - Meaning 2: $\text{id} * (\text{id} + (\text{id} * \text{id}))$
 - Meaning 3: $\text{id} * ((\text{id} + \text{id}) * \text{id})$



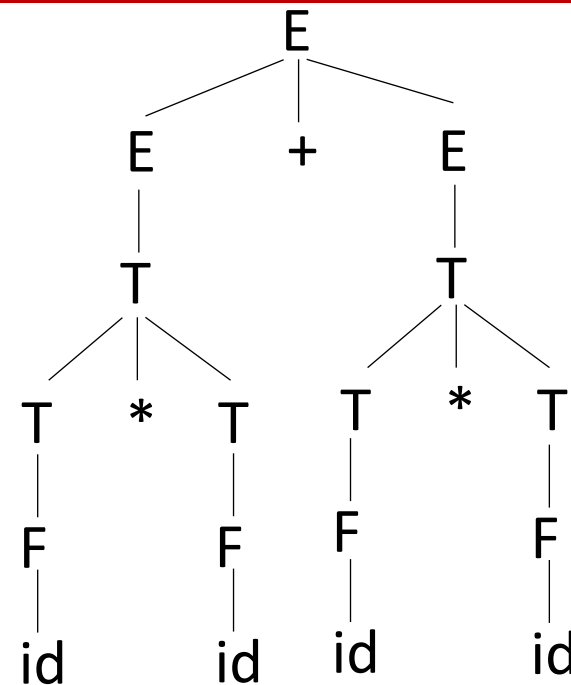
Remove Ambiguity (cont.)

$E \rightarrow E * E \mid E + E \mid (E) \mid \text{id}$

$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid \text{id}$

$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid \text{id}$

- String $\text{id} * \text{id} + \text{id} * \text{id}$ can result in
 - Meaning 1: $(\text{id} * \text{id}) + (\text{id} * \text{id})$ ✓
 - Meaning 2: $\text{id} * (\text{id} + (\text{id} * \text{id}))$
 - Meaning 3: $\text{id} * ((\text{id} + \text{id}) * \text{id})$
- String $\text{id} + \text{id} + \text{id}$



Remove Ambiguity (cont.)

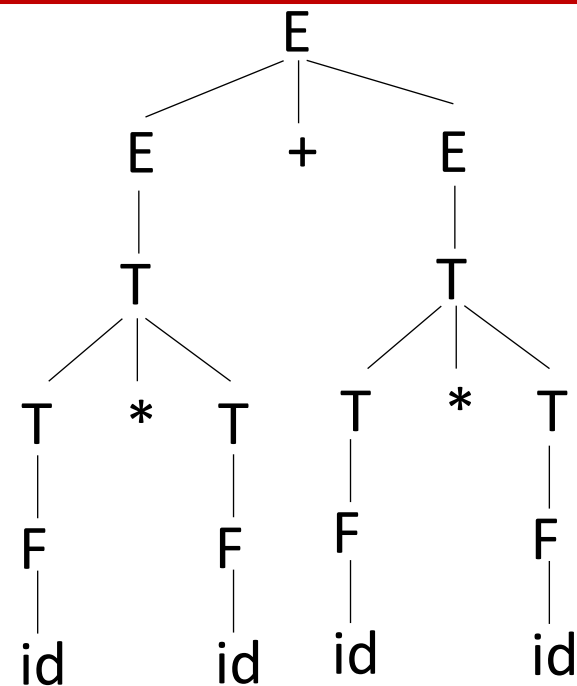
$E \rightarrow E * E \mid E + E \mid (E) \mid id$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid id$



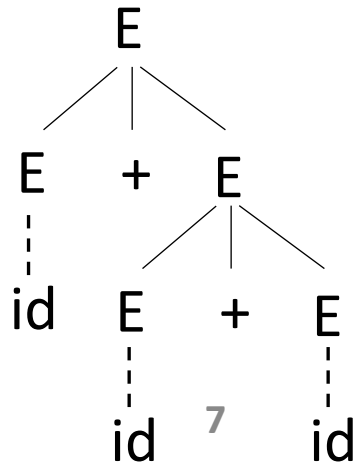
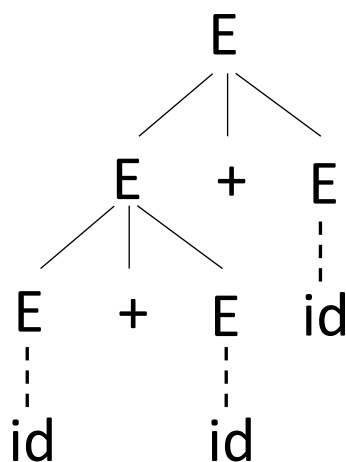
$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$



- String $id * id + id * id$ can result in

- Meaning 1: $(id * id) + (id * id)$ ✓
- Meaning 2: $id * (id + (id * id))$
- Meaning 3: $id * ((id + id) * id)$

- String $id + id + id$



Remove Ambiguity (cont.)

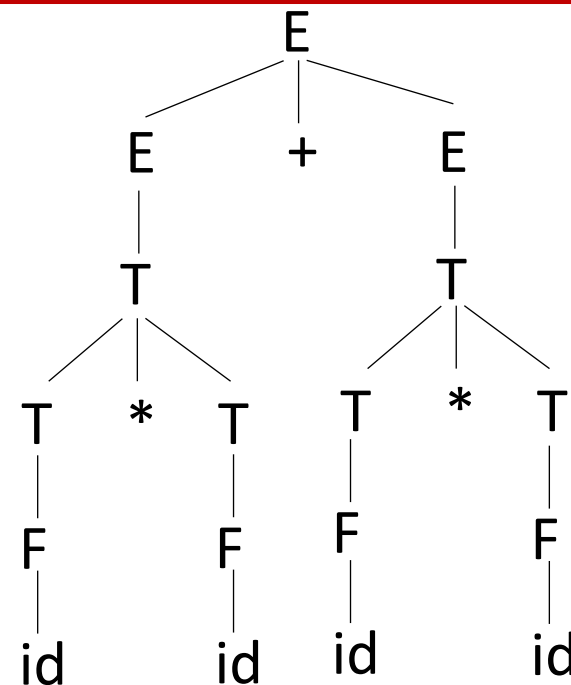
$E \rightarrow E * E \mid E + E \mid (E) \mid id$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid id$



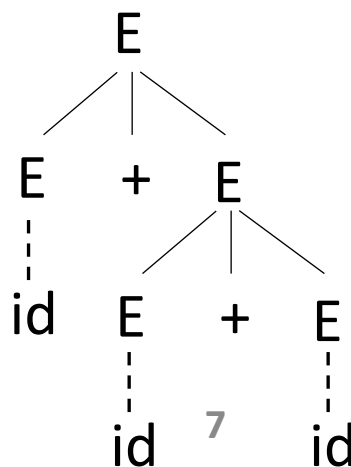
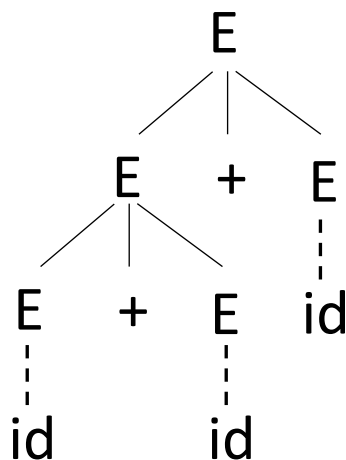
$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$



- String $id * id + id * id$ can result in

- Meaning 1: $(id * id) + (id * id)$ ✓
- Meaning 2: $id * (id + (id * id))$
- Meaning 3: $id * ((id + id) * id)$

- String $id + id + id$



Remove Ambiguity (cont.)

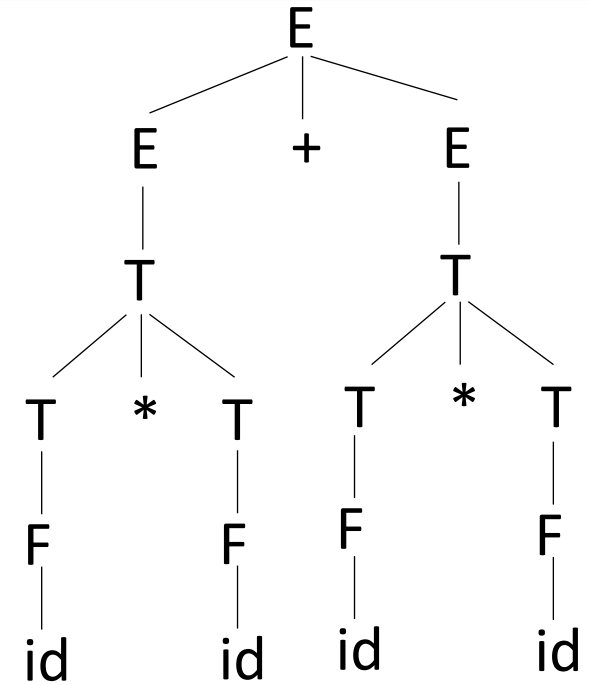
$E \rightarrow E * E \mid E + E \mid (E) \mid id$



$E \rightarrow E + E \mid T$
 $T \rightarrow T * T \mid F$
 $F \rightarrow (E) \mid id$



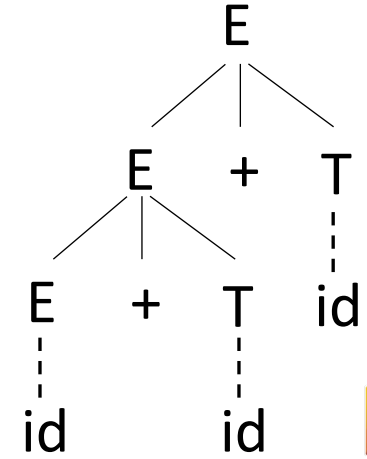
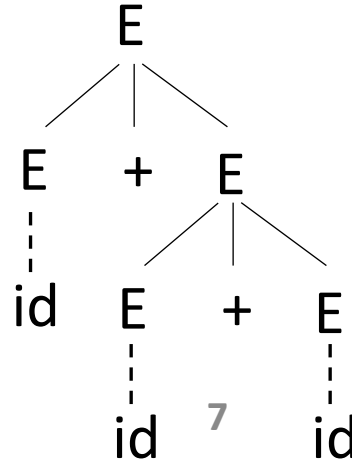
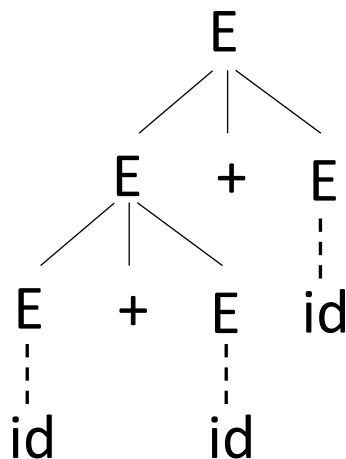
$E \rightarrow E + T \mid T$
 $T \rightarrow T * F \mid F$
 $F \rightarrow (E) \mid id$



- String $id * id + id * id$ can result in

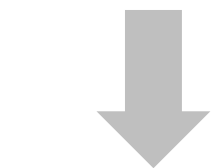
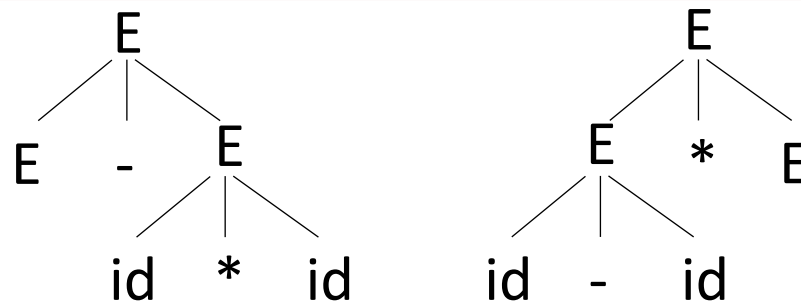
- Meaning 1: $(id * id) + (id * id)$ ✓
- Meaning 2: $id * (id + (id * id))$
- Meaning 3: $id * ((id + id) * id)$

- String $id + id + id$

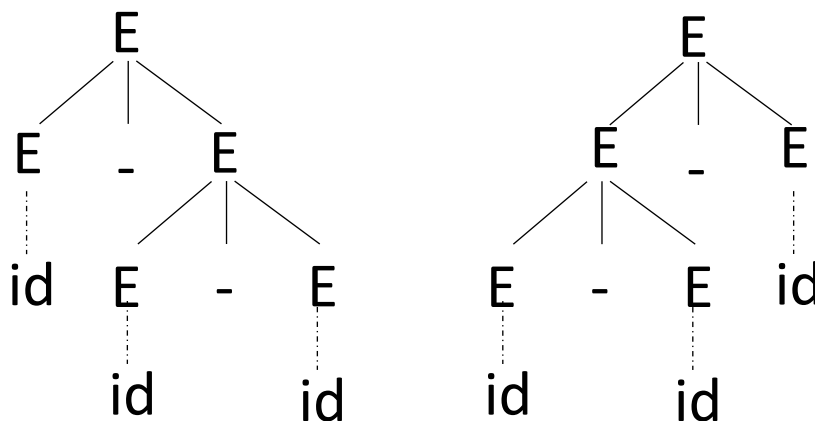


Example

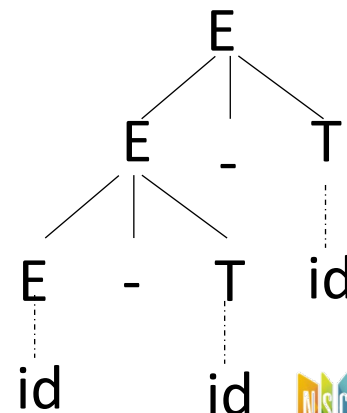
$E \rightarrow E * E \mid E - E \mid id$ //precedence: * - same
 $id - id * id$



$E \rightarrow E - E \mid T$ //precedence: * is high
 $T \rightarrow T * T \mid F$
 $F \rightarrow id$
 $id - id - id$



$E \rightarrow E - T \mid T$ //precedence: * is high
 $T \rightarrow T * F \mid F$ //associativity: - is left
 $F \rightarrow id$
 $id - id - id$



Grammar → Parser[文法到分析器]

- What exactly is **parsing**, or syntax analysis?[语法分析]
 - To process an input string for a given grammar,
 - and **compose the derivation** if the string is in the language
 - Two subtasks
 - determine if string can be derived from grammar or not
 - build a representation of derivation and pass to next phase
- What is the best representation of derivation?[推导表示]
 - Can be a parse tree or an abstract syntax tree
- An abstract syntax tree is[抽象语法树]
 - Abbreviated representation of a parse tree
 - Drops some details without compromising meaning
 - some terminal symbols that no longer contribute to semantics are dropped (e.g. parentheses)
 - internal nodes may contain terminal symbols

Example: Abstract Syntax Tree

- AST: condensed form of parse tree
 - Operators and keywords do not appear as leaves (e.g., +)
 - Chains of single productions are collapsed (e.g., $E \rightarrow T$)

G:

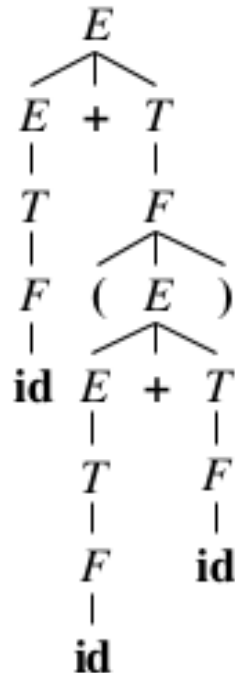
$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

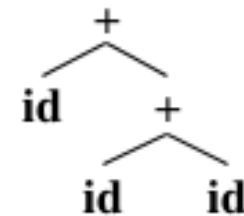
$F \rightarrow (E) \mid \text{id}$

Input:

id + (id + id)



parse tree



AST

Summary of CFG[小结]

- Compilers specify program structure using CFG
 - Most programming languages are not context free
 - Context sensitive analysis can easily be separated out to semantic analysis phase

- A parser uses CFG to
 - ... group lexical tokens to form expressions, statements, etc
 - ... answer if an input $str \in L(G)$
 - ... and build a parse tree
 - ... or build an AST instead
 - ... and pass it to the rest of compiler
 - ... or give an error message stating that str is invalid

Parser Implementation: Yacc + lex

parser.y

```
1 %{
2 #include <ctype.h>
3 #include <stdio.h>
4 #define YYSTYPE double /* double type for Yacc stack */
5 %}
6 %token NUMBER
7
8 %left '+' '-'
9 %left '*' '/'
10
11 %%
12
13 lines : lines expr '\n' { printf("= %g\n", $2); }
14       | lines '\n'
15       /* empty */
16 ;
17 expr  : expr '+' expr { $$ = $1 + $3; }
18       | expr '-' expr { $$ = $1 - $3; }
19       | expr '*' expr { $$ = $1 * $3; }
20       | expr '/' expr { $$ = $1 / $3; }
21       | '(' expr ')' { $$ = $2; }
22       | NUMBER
23 ;
24
25 E → E+E|E-E|E*E|E/E|(E)|num
26
27 /*
28 int yylex() {
29     int c;
30     while ((c = getchar()) == ' ');
31     if ((c == '.') || isdigit(c)) {
32         ungetc(c, stdin);
33         scanf("%lf", &yylval);
34         return NUMBER;
35     }
36     return c;
37 }
38 */
39
40 int main() {
41     if (yyparse() != 0)
42         fprintf(stderr, "Abnormal exit\n");
43     return 0;
44 }
45
46 int yyerror(char *s) {
47     fprintf(stderr, "Error: %s\n", s);
48 }
```

lexer.l

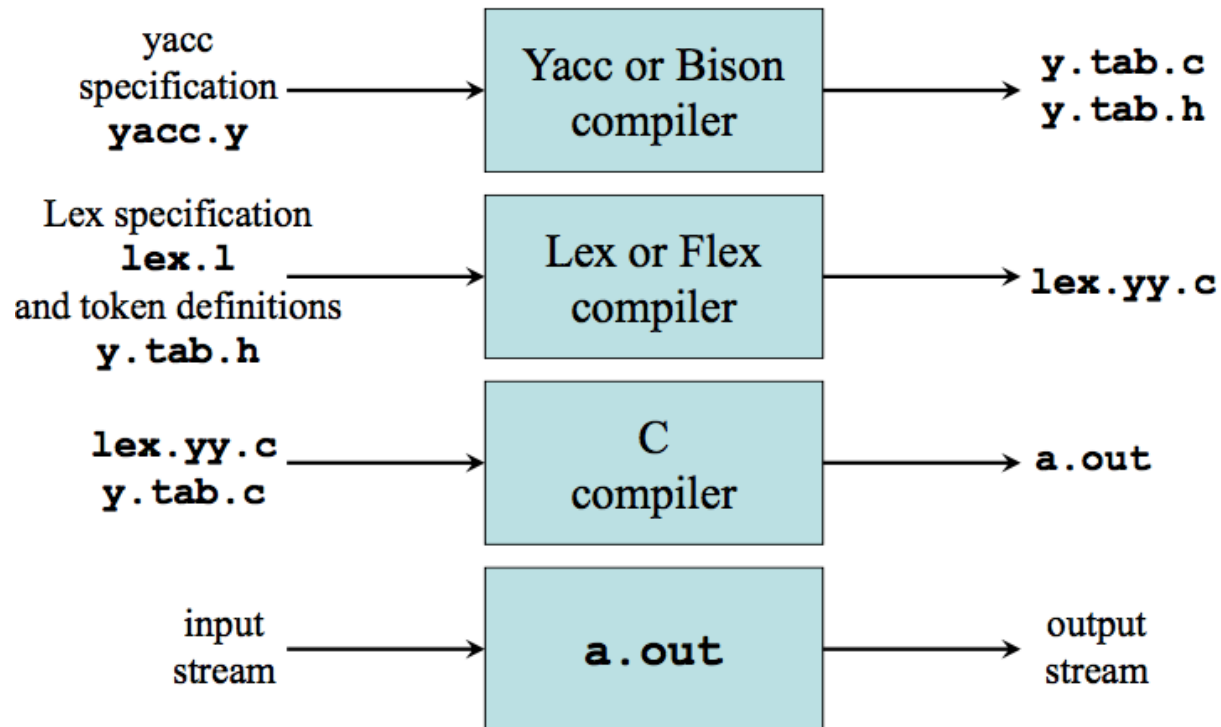
```
1 %{
2 #define YYSTYPE double
3 #include "y.tab.h"
4 extern double yyval;
5 %}
6 number [0-9]+\.[0-9]*|[0-9]*\.[0-9]+
7
8 %%
9
10 [ ]      { /* skip blanks */ }
11 {number} { sscanf(yytext, "%lf", &yylval);
12           return NUMBER; }
13 \n|.     { return yytext[0]; }
14
15 %%
16
17 int yywrap(void) {
18     return 1;
19 }
```

Generated by Yacc

Defined in y.tab.c

Yacc + Lex

- Lex was designed to produce lexical analyzers that could be used with Yacc
- Yacc generates a parser in `y.tab.c` and a header `y.tab.h`
- Lex includes the header and utilizes token definitions
- Yacc calls `yylex()` to obtain tokens



Example: Yacc + Lex (cont.)

- Compile

- `$yacc -d parser.y`
- `$lex lexer.l`
- `$clang -o test y.tab.c lex.yy.c`

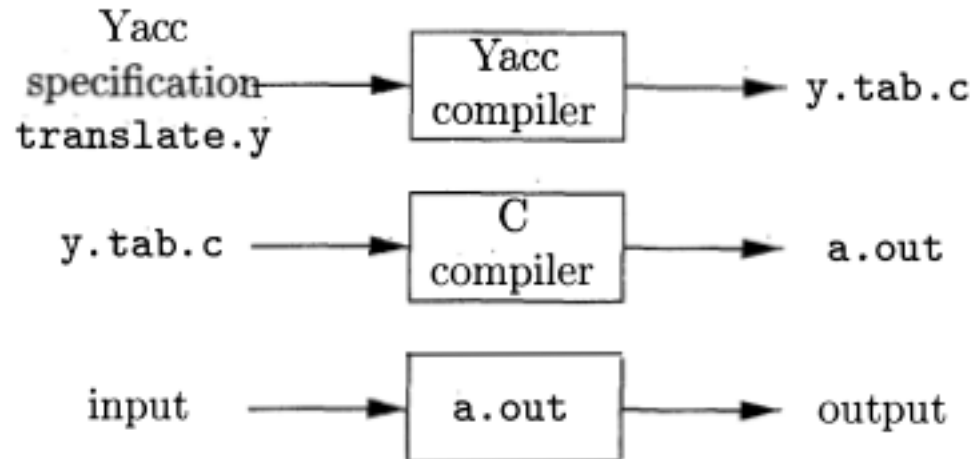
- Run

- `$/test < exprs.txt`

```
1 1 + 5
2 1 * 2 + 10
3 10 - 2 -3
```

Yacc Overview

- Yacc is an LALR(1) parser generator
 - YACC: Yet Another Compiler-Compiler
 - Parse a language described by a context-free grammar (**CFG**)
 - Yacc constructs an **LALR(1)** table
- Available as a command on the UNIX system
 - Bison: free GNU project alternative to Yacc



Yacc Specification

- **Definitions** section[定义]:
 - C declarations within `{ % }`
 - Token declarations
- **Rules** section[规则]:
 - Each rule consists of a grammar production and the associated semantic action
- **Subroutines** section[辅助函数]:
 - User-defined auxiliary functions

```
{  
  #include ...  
}  
%token NUM VAR  
%%  
production { semantic action }  
...  
%%  
...
```


Write a Grammar in Yacc

- A set of productions $\langle \text{head} \rangle \rightarrow \langle \text{body} \rangle_1 \mid \dots \mid \langle \text{body} \rangle_n$ would be written in YACC as:

```
 $\langle \text{head} \rangle : \langle \text{body} \rangle_1 \{ \langle \text{semantic action} \rangle_1 \}$   
           ...  
           :  $\langle \text{body} \rangle_n \{ \langle \text{semantic action} \rangle_n \}$   
           ;
```

- Usages

- Tokens that are single characters can be used directly within productions, e.g. '+'
- Named tokens must be declared first in the declaration part using `%token TokenName`

Write a Grammar in Yacc (cont.)

- Semantic actions may refer to values of the synthesized attributes of terminals and non-terminals in a production:

$X : Y_1 Y_2 Y_3 \dots Y_n \{ \text{action} \}$

- $\$ \$$ refers to the value of the attribute of X (non-terminal)
 - $\$ i$ refers to the value of the attribute of Y_i (terminal or non-terminal)
 - Normally the semantic action computes a value for $\$ \$$ using $\$ i$'s
- Example: $E \rightarrow E + T \mid T$
expr : expr '+' term { $\$ \$ = \$ 1 + \$ 2$ }

| term

;

default action: { $\$ \$ = \$ 1$ }

Example: $E \rightarrow E+E | E-E | E * E | E/E | (E) | \text{num}$

```
1 %{
2 #include <ctype.h>
3 #include <stdio.h>
4 #define YYSTYPE double /* double type for Yacc stack */
5 %}
6 %token NUMBER
7
8 %left '+' '-'
9 %left '*' '/'
10
11 %%
12
13 lines : lines expr '\n' { printf("= %g\n", $2); }
14       | lines '\n'
15       | /* empty */
16       ;
17 expr  : expr '+' expr { $$ = $1 + $3; }
18       | expr '-' expr { $$ = $1 - $3; }
19       | expr '*' expr { $$ = $1 * $3; }
20       | expr '/' expr { $$ = $1 / $3; }
21       | '(' expr ')' { $$ = $2; }
22       | NUMBER
23       ;
```

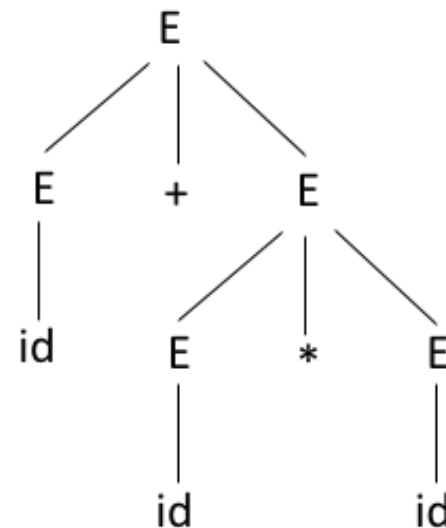
Can we remove those two lines?

Allow to evaluate a sequence of expressions, one to a line

ϵ

Parser Types[分析器类型]

- **Grammar** is used to derive string or construct **parser**
- Most compilers use either **top-down** or **bottom-up** parsers
- **Top-down parsing**[自顶向下分析]
 - Starts from root and expands into leaves
 - Tries to expand start symbol to input string
 - Finds leftmost derivation[最左推导]
 - In each step
 - Which non-terminal to replace?
 - Which production of the non-terminal to use?
 - Parser code structure closely mimics grammar
 - Amenable to implementation by hand
 - Automated tools exist to convert to code (e.g. ANTLR)



Parser Types (cont.)

- Bottom-up parser[自底向上分析]
 - Starts at leaves and builds up to root
 - Tries to reduce the input string to the start symbol
 - Finds reverse order of the rightmost derivation[最右推导的逆 → 最左归约, 也称为规范归约]
 - Parser code structure nothing like grammar
 - Very difficult to implement by hand
 - Automated tools exist to convert to code (e.g. Yacc, Bison)
 - $LL \subset LR$ (Bottom-up works for a larger class of grammars)
- Top-down vs. bottom-up[对比]
 - Top-down: easier to understand and implement manually
 - E.g., ANTLR
 - Bottom-up: more powerful, can be implemented automatically
 - E.g., YACC/Bison

Example

- Consider a CFG grammar G

$$S \rightarrow AB$$

$$A \rightarrow aC$$

$$B \rightarrow bD$$

$$D \rightarrow d$$

$$C \rightarrow c$$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$$\begin{aligned} S &\Rightarrow AB \quad (1) \\ &\Rightarrow aCB \quad (2) \\ &\Rightarrow acB \quad (3) \\ &\Rightarrow acbD \quad (4) \\ &\Rightarrow acbd \quad (5) \end{aligned}$$

Bottom-up (reverse of rightmost derivation)

$$\begin{aligned} S &\Rightarrow AB \quad (5) \\ &\Rightarrow AbD \quad (4) \\ &\Rightarrow Abd \quad (3) \\ &\Rightarrow aCbd \quad (2) \\ &\Rightarrow acbd \quad (1) \end{aligned}$$

Example

- Consider a CFG grammar G

$$S \rightarrow AB$$

$$A \rightarrow aC$$

$$B \rightarrow bD$$

$$D \rightarrow d$$

$$C \rightarrow c$$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$$\begin{aligned} S &\Rightarrow AB \quad (1) \\ &\Rightarrow aCB \quad (2) \\ &\Rightarrow acB \quad (3) \\ &\Rightarrow acbD \quad (4) \\ &\Rightarrow acbd \quad (5) \end{aligned}$$

Bottom-up (reverse of rightmost derivation)

$$\begin{aligned} S &\Rightarrow AB \quad (5) \\ &\Rightarrow AbD \quad (4) \\ &\Rightarrow Abd \quad (3) \\ &\Rightarrow aCbd \quad (2) \\ &\Rightarrow acbd \quad (1) \end{aligned}$$

S

Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

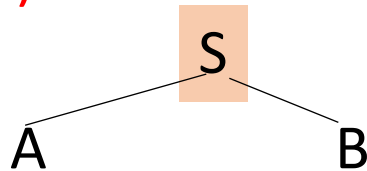
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

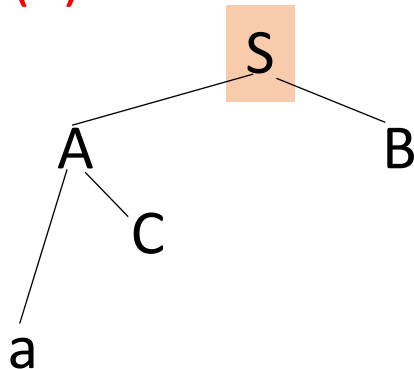
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

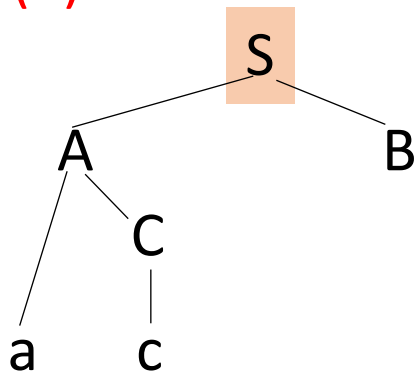
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

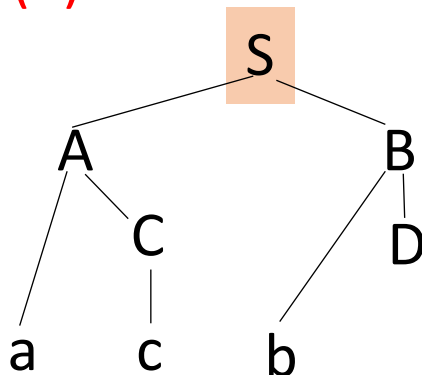
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

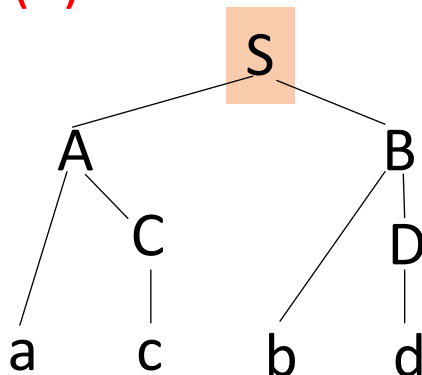
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

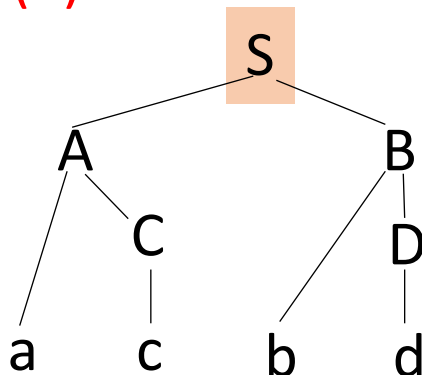
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

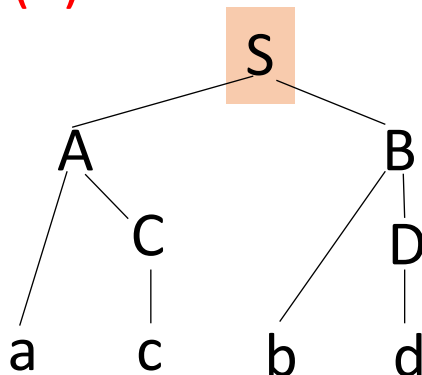
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

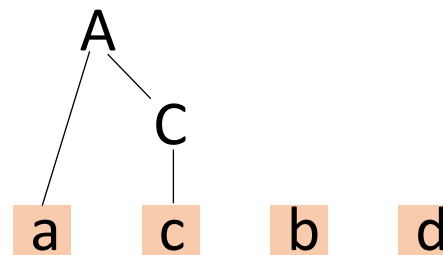
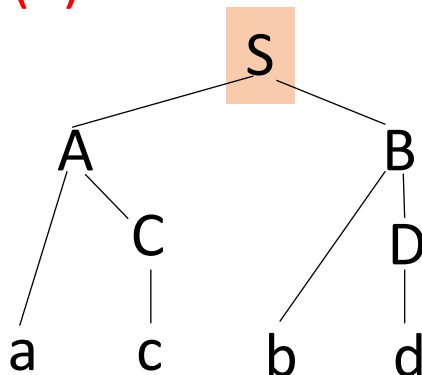
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

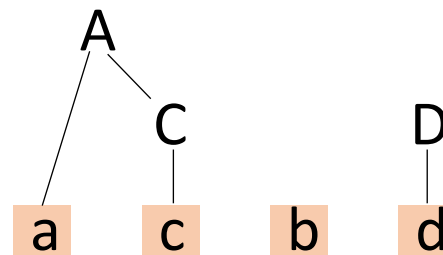
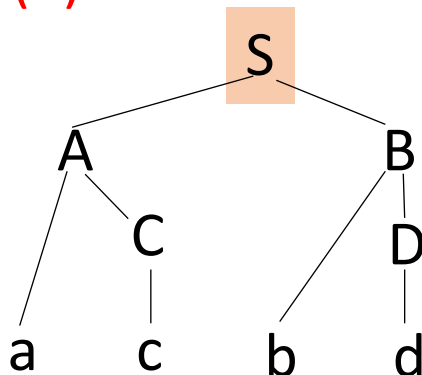
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)

Bottom-up (reverse of rightmost derivation)

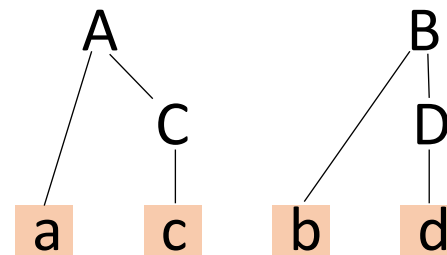
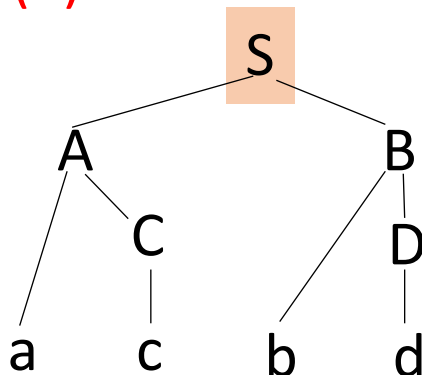
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Example

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

- This language has only one sentence: $L(G) = \{acbd\}$

Top-down (leftmost derivation)

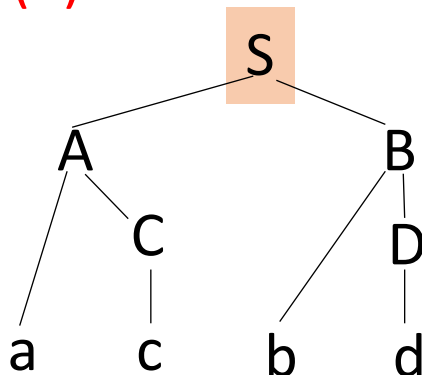
$S \Rightarrow AB$ (1)

$\Rightarrow aCB$ (2)

$\Rightarrow acB$ (3)

$\Rightarrow acbD$ (4)

$\Rightarrow acbd$ (5)



Bottom-up (reverse of rightmost derivation)

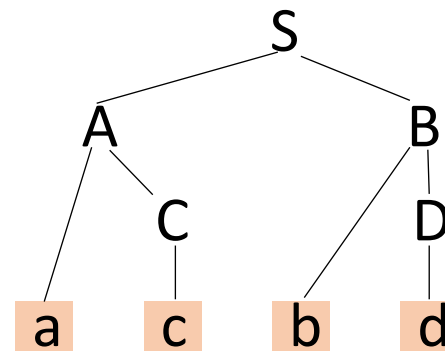
$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)



Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)

Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	<u>SUCCESS!</u>

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)

Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	<u>SUCCESS!</u>

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)

Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

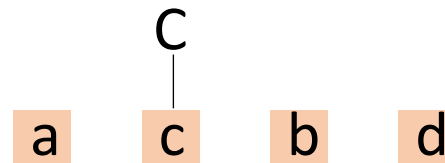
$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	SUCCESS!

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)



Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

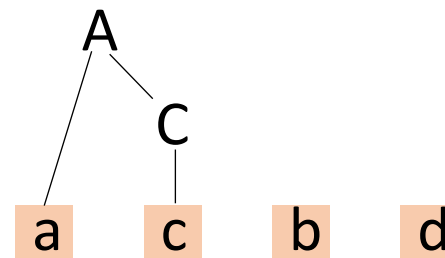
$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	SUCCESS!

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)



Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

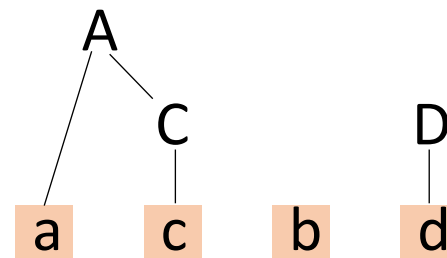
$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	SUCCESS!

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)



Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

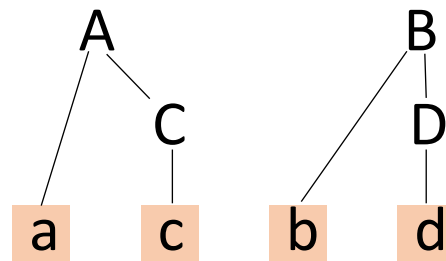
$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	SUCCESS!

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)
 $\Rightarrow AbD$ (4)
 $\Rightarrow Abd$ (3)
 $\Rightarrow aCbd$ (2)
 $\Rightarrow acbd$ (1)



Preview: Bottom-up Parsing[自低向上]

- Consider a CFG grammar G

$S \rightarrow AB$

$A \rightarrow aC$

$B \rightarrow bD$

$D \rightarrow d$

$C \rightarrow c$

Stack	Input	Action
\$	acbd\$	Shift
\$a	cbd\$	Shift
\$ac	bd\$	Reduce
\$aC	bd\$	Reduce
\$A	bd\$	Shift
\$Ab	d\$	Shift
\$Abd	\$	Reduce
\$AbD	\$	Reduce
\$AB	\$	Reduce
\$S	\$	SUCCESS!

Bottom-up (reverse of rightmost derivation)

$S \Rightarrow AB$ (5)

$\Rightarrow AbD$ (4)

$\Rightarrow Abd$ (3)

$\Rightarrow aCbd$ (2)

$\Rightarrow acbd$ (1)

